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14. ABSTRACT The goal of this project is to fully develop Banach space methods for kernel-based machine learning that extend the Hilbert space framework of regularized learning. We proposed to study Reproducing Kernel Banach Spaces (RKBS) by the semi-inner-product, develop the theory of vector-valued RKBS with applications of RKBS to manifold learning, study frames and Riesz bases for sequence spaces, and construct RKBS with the $l_1$ -norm known to enforce sparse solutions. We will also explore classification algorithms that are mathematically rigorous while rooted in human cognitive principles for categorization. Our execution plan includes three specific topics ("Aims")					
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## Report Title

Final Report: Semi-inner-products in Banach Spaces with Applications to Regularized Learning, Sampling, and Sparse Approximation

### ABSTRACT

The goal of this project is to fully develop Banach space methods for kernel-based machine learning that extend the Hilbert space framework of regularized learning. We proposed to study Reproducing Kernel Banach Spaces (RKBS) by the semi-inner-product, develop the theory of vector-valued RKBS with applications of RKBS to manifold learning, study frames and Riesz bases for sequence spaces, and construct RKBS with the  $l_1$ -norm known to enforce sparse solutions. We will also explore classification algorithms that are mathematically rigorous while rooted in human cognitive principles for categorization. Our execution plan include three specific topics (“Aims”) 1. Apply RKBS theory to Orlicz space, to perform convergence analysis, and to study Shannon sampling schemes; 2. Work out vector-valued RKBS, and study s.i.p with  $l_1$  norm; 3. Develop frames and Riesz bases for Banach spaces, and extend analysis and synthesis operators.

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**Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

<u>Received</u>	<u>Paper</u>
01/30/2015	5.00 Jun Zhang, University of Michigan, Ann Arbor. Nonparametric information geometry: From divergence function to referential-representational biduality in statistical manifolds, Entropy, (12 2013): 5384. doi:
01/31/2015	6.00 Roman Ilin, Jun Zhang, Leonid Perlovsky, Robert Kozma. Vague-to-crisp dynamics of percept formation modeled as operant (selectionist) process, Cognitive Neurodynamics, (08 2013): 0. doi: 10.1007/s11571-013-9262-0
03/31/2016	10.00 Jun Zhang. On Monotone Embedding in Information Geometry, Entropy, (06 2015): 0. doi: 10.3390/e17074485
03/31/2016	9.00 Haizhang Zhang, Jun Zhang. Regularized learning in Banach spaces as an optimization problem: representer theorems, Journal of Global Optimization, (07 2010): 0. doi: 10.1007/s10898-010-9575-z
11/03/2013	2.00 Jun Zhang, Haizhang Zhang. Vector-valued reproducing kernel Banach spaces with applications to multi-task learning, Journal of Complexity, (04 2013): 0. doi: 10.1016/j.jco.2012.09.002
<b>TOTAL:</b>	<b>5</b>

Number of Papers published in peer-reviewed journals:

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(b) Papers published in non-peer-reviewed journals (N/A for none)

Received      Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

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(c) Presentations

Number of Presentations: 0.00

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Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received      Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

<u>Received</u>	<u>Paper</u>
01/30/2015	3.00 Praveen K. Yenduri, Anna C. Gilbert, Jun Zhang. Integrate-and-fire neuron modeled as a low-rate sparse time-encoding device, 2012 Third International Conference on Intelligent Control and Information Processing (ICICIP). 15-JUL-12, Dalian, China. : ,
01/30/2015	4.00 Praveen K. Yenduri, Anna C. Gilbert. Compressive, collaborative spectrum sensing for wideband Cognitive Radios, 2012 9th International Symposium on Wireless Communication Systems (ISWCS 2012). 28-AUG-12, Paris, France. : ,
03/30/2016	7.00 Roman Ilin, Jun Zhang. Information fusion with uncertainty modeld on topological event spaces, Symposium on the Foundation of Computation Intelligence FOCI'2014. 09-DEC-14, . : ,
03/30/2016	8.00 Roman Ilin, Jun Zhang. Information fusion with topological event spaces, 18th International Conference on Information Fusion Washington DC. 06-JUL-15, . : ,
TOTAL:	4

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

<u>Received</u>	<u>Paper</u>
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TOTAL:

Number of Manuscripts:

Books

<u>Received</u>	<u>Book</u>
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TOTAL:

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Yinbin Lei	1.00
FTE Equivalent:	1.00
Total Number:	1

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

### Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ..... 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: ..... 0.00

### Names of Personnel receiving masters degrees

NAME

**Total Number:**

### Names of personnel receiving PHDs

NAME

**Total Number:**

### Names of other research staff

NAME

PERCENT SUPPORTED

**FTE Equivalent:**

**Total Number:**

### Sub Contractors (DD882)

### Inventions (DD882)

### Scientific Progress

### Technology Transfer

Interaction with AFRL scientist Roman Ilin (sensor directorate), which led to several publications

# ARO Final Report

Semi-inner-products in Banach spaces with applications  
to regularized learning, sampling, and sparse approximations

Contract/grant number: W911NF-12-1-0163

Author of the report: Jun Zhang (PI)

Performing organization: The University of Michigan  
530 Church Street, Ann Arbor, MI 48109

ARO proposal number: No. 61819-MA

## Project Abstract

The goal of this project is to fully develop Banach space methods for kernel-based machine learning that extend the Hilbert space framework of regularized learning. We proposed to study Reproducing Kernel Banach Spaces (RKBS) by the semi-inner-product, develop the theory of vector-valued RKBS with applications of RKBS to manifold learning, study frames and Riesz bases for sequence spaces, and construct RKBS with the  $l^1$ -norm known to enforce sparse solutions. We will also explore classification algorithms that are mathematically rigorous while rooted in human cognitive principles for categorization. Our execution plan include three specific topics (“Aims”) 1. Apply RKBS theory to Orlicz space, to perform convergence analysis, and to study Shannon sampling schemes; 2. Work out vector-valued RKBS, and study s.i.p with  $l^1$  norm; 3. Develop frames and Riesz bases for Banach spaces, and extend analysis and synthesis operators.

## Accomplishments

1. Motivated by multi-task machine learning with Banach spaces, we introduced the notion of vector-valued reproducing kernel Banach spaces (RKBS), and investigated basic properties of the spaces and the associated reproducing kernels. As a highlight, we proved that the kernel  $K(x,y)$  in a vector-valued RKBS is a bounded operator satisfying  $K(x,y)=\Phi(x)\Phi^T(y)$  where  $\Phi()$  is an operator-valued feature map, with  $\Phi^T()$  its generalized adjoint, and  $x,y$  are sample points. This is an extension of the feature map in (scalar-valued) RKBS case. Several concrete examples of the construction of the feature map in vector-valued RKBS were presented. Applying our framework to multi-task learning, we established the representer theorem and characterization equations for the minimizer of regularized learning schemes in vector-valued RKBS.

2. Frame is an extension of the notion of base in a vector space, by allowing dependence among members of a collection of vectors. A Riesz base is the image, under an invertible linear transformation, of orthonormal base in a Hilbert space. Traditionally, when extending those concepts to the Banach space  $B$ , their definitions involve elements of the dual space  $B^*$ . We used the notion of semi-inner-product and propose to define them as a collection of elements in  $B$  directly. Generalizing the classical theory of Hilbert space frames and Riesz basis under this new perspective, we then established a corresponding Shannon sampling theorem in Banach spaces.

3. We characterized Orlicz space semi-inner product reproducing kernel. We also derived the s.i.p with  $l^1$  norm, and made progress about constructing reproducing kernels  $l^1$  norm based on a priori given kernel function, but a satisfactory application of theory to derive s.i.p reproducing kernel remains a challenge.

4. Based on compressed sensing framework, we were able to propose and construct a low-rate spiking neuron which exploits the sparsity or compressibility present in natural signals. Our model neuron belongs to the class of IAF (Integrate-and-Fire) neuron model; however we provided appropriate modifications to its dead-time ("absolute refractory period" in neuronal term) to convert it into a low-rate encoder, and develop an algorithm for reconstructing/decoding the input stimulus from the low-rate spike trains. Our neuron produces spikes at a firing rate proportional to the amount of information present in the signal rather than its duration. Through simulations with frequency-sparse signals, we demonstrated superior performance of our Low-Rate IAF neuron operating at a sub-Nyquist rate, when compared with state-of-the-art IAF neurons proposed earlier (e.g., A. Lazar's Time Encoding Machines or TEMs), which operate at and above Nyquist rates.

5. Another direction (not in the original plan) that we have worked and made major progress is a formal framework to unifying feature learning and generalization in cross-table with a list of objects (in rows) and a list of features (in columns) and derive the so-called "concept lattice" - a lattice is a partially-ordered set closed in "meet" and "join" operations. We are able to reformulate this existing framework as "subset system", or a system of subsets on a set. For any subset system, we show that, in addition to the two natural relations of subsets involved (namely, inclusion and non-zero intersection), there are two induced relations on elements of the base-set (which model, respectively, \*similarity\* by a symmetric but non-transitive relation and \*specificity\* by a transitive and asymmetric pre-order relation). We can define "neighborhood" and "separation" in ways extending those as used in abstract topology. This provides a mathematically concise way of modeling "context" in a relational data.

## **Training Activity**

Finally, during the summer of 2015, the PI ran a summer REU (Research Experience in Undergraduate) program where advanced undergraduate students were brought



to University of Michigan for 2-8 weeks to engage research projects conducted by the M3 Lab: Alan Aw (Stanford University), Mark Greenfield (Caltech), Luciana Xiao (University of Notre Dame), Bradley Zykoski (University of Virginia), Michael Lewis (New Mexico Institute of Mining Technology). This was the first time the PI engages undergraduate students in scale, and was a valuable experience in training and exposing research questions to the next generation of mathematician and scientists.